CONVERTING HAZUS CAPACITY CURVES TO SEISMIC HAZARD-COMPATIBLE BUILDING FRAGILITY FUNCTIONS: EFFECT OF HYSTERETIC MODELS

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ABSTRACT:

A methodology was recently proposed for the development of hazard-compatible building fragility models using parameters of capacity curves and damage state thresholds from HAZUS (Karaca and Luco, 2008). In the methodology, HAZUS curvilinear capacity curves were used to define nonlinear dynamic SDOF models that were subjected to the nonlinear time history analysis instead of the capacity spectrum method. In this study, we construct a multilinear capacity curve with negative stiffness after an ultimate (capping) point for the nonlinear time history analysis, as an alternative to the curvilinear model provided in HAZUS. As an illustration, here we propose parameter values of the multilinear capacity curve for a moderate-code low-rise steel moment resisting frame building (labeled S1L in HAZUS). To determine the final parameter values, we perform nonlinear time history analyses of SDOF systems with various parameter values and investigate their effects on resulting fragility functions through sensitivity analysis. The findings improve capacity curves and thereby fragility and/or vulnerability models for generic types of structures.

KEYWORDS: HAZUS, Capacity curve, Fragility function

1. INTRODUCTION

Fragility functions for generic structural models are very useful for estimating seismic risk on a regional level in a relatively simple manner, but are of course less accurate than building-specific fragility functions for a particular building. HAZUS, a popular risk assessment tool, has fragility functions for a total of 36 generic structural models and 4 design code levels (High-Code, Moderate-Code, Low-Code, and Pre-Code). However, HAZUS fragility functions are not in a format that can be coupled with hazard curves (e.g., USGS hazard curves) for a fully-probabilistic risk assessment since they are not conditioned on spectral acceleration; instead they are conditioned on building response. Thus, Karaca and Luco (2008) recently proposed a methodology for the development of hazard-compatible building fragility models using parameters of capacity curves and damage state thresholds from HAZUS.

In the methodology of Karaca and Luco (2008), building response was estimated by time history analysis of single degree of freedom systems corresponding to the HAZUS pushover curves under a large number of earthquake records, instead of the capacity spectrum method applied in HAZUS. The resulting fragility functions conditioned on a single scalar spectral acceleration are derived using statistics of the building response as a function of spectral acceleration, together with damage state thresholds from HAZUS that are in terms of building response (expressed in terms of inelastic spectral displacements). Hazard-compatible fragility functions can then be coupled with existing hazard information, such as USGS seismic hazard curves. Also, the methodology allows one to account for, in derivation of the fragility functions, uncertainties in the building
capacity curve and damage state thresholds, as well as ground motion record-to-record variability in building response.

Karaca and Luco (2008) used the curvilinear capacity curves provided in HAZUS (see Figure 2 for an example) in order to be consistent. However, those curves were intended to be used for the capacity spectrum method, rather than for nonlinear time history analysis. So we may improve hazard-compatible building fragility models if we choose a more widely available and flexible capacity curve parameterization. As shown in Figure 1, in this study we use multilinear capacity curves (see Figure 3 for an illustration) instead of the curvilinear capacity curves provided in HAZUS, due to the following reasons: 1) There are many available structural analysis programs using multilinear back bones (e.g., OpenSees). In those programs, we can implement different hysteresis models such as pinching or Clough models. 2) With multilinear capacity curves we can introduce negative stiffness past the ultimate (capping) point, which can have significant effects on the response in nonlinear dynamic analyses (Ibarra, 2003). With negative post-capping stiffness and various hysteresis models, one can simulate strength and/or stiffness deterioration and collapse behavior.

In this study, we review the HAZUS curvilinear capacity curves and point out limitations if we use them for nonlinear time history analysis. We then construct multilinear capacity curves with negative stiffness after an ultimate (capping) point, as an alternative to the HAZUS curvilinear curves. We propose appropriate capacity parameter values for the multilinear capacity curves, as an illustration in this paper, for a moderate-code low-rise steel moment resisting frame (labeled S1L in HAZUS). To determine the final capacity parameter values, we perform nonlinear time history analyses of SDOF systems with various parameter values and investigate their effects on resulting fragility functions through sensitivity analysis. The findings improve capacity curves and thereby fragility and/or vulnerability models for generic types of structures.

2. HAZUS CAPACITY CURVE

Each HAZUS building capacity curve is a plot of spectral displacement and acceleration converted from a static pushover curve of base shear versus displacement. It is intended to be used in the capacity spectrum method rather than nonlinear time history analysis. Figure 2 shows the curvilinear HAZUS capacity curve for a moderate-code low-rise steel moment resisting frame (S1L) building. The building capacity curve consists of two control points: yield and ultimate capacity. The yield capacity represents the elastic lateral strength of the building considering conservatisms in design. The ultimate capacity represents the maximum strength of the building when the global system has reached a fully plastic state. The capacity curve is linear up to the yield point, it transitions in slope from an elastic state to a fully plastic state from the yield point to ultimate point, and it remains plastic past the ultimate point.
The yield capacity acceleration, \( A_y \), is computed, as shown in Eqn. 2.1, using a design strength coefficient, \( C_y \), that is approximately based on the lateral-force design requirements of current seismic design codes (e.g., 1994 NEHRP Provisions). The conversion from yield acceleration, \( A_y \), to yield displacement, \( D_y \), is computed using the elastic fundamental-mode period of the building, \( T_e \).

\[
A_y = C_y \gamma / \alpha_i \quad (g) \\
D_y = 9.8 A_y T_e^2 \quad (in)
\]  

(2.1)

where \( \gamma \) is the overstrength factor relating yield strength to design strength and \( \alpha_i \) is the fraction of building weight effective in the push-over mode.

The ultimate capacity acceleration and displacement are computed, as shown in Eqn. 2.2, by multiplying the yield capacity acceleration and displacement with, respectively, an overstrength factor, \( \lambda \), and the multiplication of the overstrength factor, \( \lambda \), and ductility factor, \( \mu \). For the example S1L building, the elastic fundamental-mode period (\( T_e \)) is 0.5 sec, the ductility factor (\( \mu \)) is 6, and the overstrength factor (\( \mu \)) is 3. It should be noted that the building period (\( T_e \)), push-over mode parameter (\( \alpha_i \)), and overstrength factors (\( \gamma \) and \( \lambda \)) are assumed to be independent of design code level, while the design strength coefficient (\( C_y \)) and ductility factor (\( \mu \)) are dependent on both building type and design level.

\[
A_u = \lambda A_y \quad (g) \\
D_u = \lambda \mu D_y \quad (in)
\]  

(2.2)

The capacity curve parameters are based on a combination of engineering calculations and judgment (Kircher 2007, personal communication). Since they are originally derived from static pushover curves and converted for the capacity spectrum method, they may not be fully suitable for nonlinear time history analyses. In particular, the curve is assumed to have plastic behavior without any strength and/or stiffness deterioration past the ultimate point, which is not realistic for general buildings. Also, the ratio of ultimate and yield displacement, i.e., the effective ductility, is too large for real structures (e.g., 18 for the example S1L building). This is because the ultimate capacity displacement is not the “true” ultimate displacement capacity of the system; it is just a point along the capacity curve at which maximum strength has been fully attained (Kircher 2007, personal communication). To overcome these limitations and to arrive at multilinear capacity curves appropriate for nonlinear time history analyses, we need to determine parameter values of multilinear capacity curves. In doing so, we investigate the effect of varying capacity parameter values on fragility functions, as described in the next section.
3. MULTILINEAR CAPACITY CURVE

We construct a multilinear capacity curve for nonlinear time history analysis as an alternative to the curvilinear capacity curve provided in HAZUS due to the reasons described in the Section 1. The multilinear capacity curve consists of three points: yield, ultimate, and residual capacity, as shown in Figure 3. In addition to adjusting the yield \((D_y^*, A_y^*)\) and ultimate points \((D_u^*, A_u^*)\) that already exist in the HAZUS curvilinear curve, we introduce a residual capacity point \((D_r^*, A_r^*)\), and thereby negative stiffness past the ultimate (capping) point, because it has significant effects on the responses in nonlinear dynamic analyses (Ibarra, 2003).

Figure 3 Multilinear capacity curve

Figure 4 shows a tree illustrating the options we considered for determining the multilinear capacity curve parameters, where the chosen options are underlined. In determining the parameter values, we perform nonlinear time history analyses of SDOF systems with various capacity parameter values and investigate their effects on resulting fragility functions through sensitivity analysis. The SDOF systems with the multilinear capacity curves are subjected to a suite of 1554 ground motions from the PEER Next Generation Attenuation (NGA) database. We construct the fragility functions by combining building response and its variability from regression analysis for nonlinear displacement demand, with damage state thresholds provided in HAZUS. A more detailed description of the input ground motions, regression analysis procedure, and construction of fragility functions can be found in Karaca and Luco (2008).

![Figure 4 A tree illustrating options for determining multilinear capacity curve parameters](attachment:image.png)
For each multilinear capacity curve parameter, we make a decision considering the concepts behind HAZUS capacity curves, which we reviewed in Section 2, and the effects of varying the parameter values on the fragility functions. As an illustration, here we determine parameter values for a moderate-code low-rise steel moment resisting frame building (labeled S1L in HAZUS). We use hereafter “Curvilinear” to denote the curvilinear capacity curve with the parameter values provided in HAZUS, “Trilinear” to denote the multilinear curve without the residual capacity point (i.e., zero stiffness after the ultimate point), and “Multilinear” to denote the multilinear curve with the residual capacity point.

3.1. Yield Capacity Point (\(D_y^*\) and \(A_y^*\))

A yield capacity point consistent with HAZUS may be determined in the two ways shown in Figure 4: 1) using the HAZUS yield capacity point; 2) computing a new yield capacity using the equal area rule that is often used to estimate the yield point of a bilinear capacity curve that approximates a curvilinear one. Note that in our application of the equal area rule we maintain the initial stiffness of the structure.

Figure 5 shows trilinear capacity curves with both yield points and the corresponding HAZUS curvilinear curve for the example S1L building. An advantage of using the HAZUS yield point (denoted \((D_y, A_y)\) in Figure 5) is that it is based on a combination of seismic design codes and expert judgment, as described in the Section 2. However, using \(D_y\) results in an effective ductility and strain hardening ratio (e.g., 18 and 0.12 respectively for the example S1L building) that are unreasonably large for general buildings. Using the equal area rule shifts the yield displacement (denoted \((D_y^*, A_y^*)\) in Figure 5) in a way that makes the ductility and strain hardening ratio more appropriate for general buildings.

![Figure 5 Capacity curves with various yield points](image)

Table 1 summarizes the median spectral acceleration capacity from the fragility function for each damage state (i.e., \(S_a(T = 0.5\,\text{sec})\) corresponding to a damage state exceedance probability of 50%) that is computed for the two trilinear capacity curves and the curvilinear one. Note that the median spectral acceleration is not computed directly from the capacity curve, but from the methodology described briefly in Section 1. It shows that, in the slight and moderate damage states, the trilinear model with a new value based on the equal area rule is as close as or closer to the curvilinear curve than the one with the original HAZUS value. For the extensive and complete damage states, we don’t expect the sensitivity observed in Table 1 for the multilinear capacity curve we will eventually arrive at. Thus we choose the yield capacity point based on the equal area rule for the new yield capacity point.
Table 1 Median spectral acceleration capacity for each damage state for the trilinear curves with two different yield capacity points compared with that for the curvilinear curve with the HAZUS parameter values

<table>
<thead>
<tr>
<th>Damage states</th>
<th>Trilinear with ((D_y, A_y))</th>
<th>Trilinear with ((D_y^<em>, A_y^</em>))</th>
<th>Curvilinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slight</td>
<td>0.55</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.84</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>Extensive</td>
<td>1.63</td>
<td>1.77</td>
<td>1.68</td>
</tr>
<tr>
<td>Complete</td>
<td>3.46</td>
<td>3.81</td>
<td>3.52</td>
</tr>
</tbody>
</table>

3.2. Ultimate Capacity Point \((D_u^* and A_u^*)\)

Given the new yield capacity point determined in Section 3.1, the ultimate capacity displacement may be determined in the three ways shown in Figure 4: 1) \(\mu \times D_y\), multiplication of the HAZUS ductility factor, \(\mu\), and the HAZUS yield capacity displacement, \(D_y\); 2) \(D_u = \lambda \times \mu \times D_y\), use of the HAZUS ultimate capacity displacement; 3) \(\mu \times D_y^*\), multiplication of the HAZUS ductility factor, \(\mu\), and the new yield capacity displacement, \(D_y^*\).

Presuming we choose to use the HAZUS ultimate yield acceleration, which is an option discussed at the end of this subsection, Figure 6 shows trilinear capacity curves with the three ultimate displacements for the example S1L building. The effective ductility values (i.e., ultimate displacement divided by yield displacement) for the three options and the example S1L building are 2.76, 8.15, and 6.0, respectively. The first option \(\mu \times D_y\) results in an unreasonably small ductility and unreasonably large strain hardening ratio for general buildings. Also, the resulting ultimate capacity displacement is unreasonably small compared to the median damage state threshold value from HAZUS for the extensive or complete damage states, as shown in Figure 6. If we choose the second option \(D_u\), then the effective ductility is comparable to the original ductility factor, but \(D_u\) from the HAZUS curvilinear capacity curve does not have a solid reason to be maintained for nonlinear time history analysis (in place of the capacity spectrum method), as described in Section 2. Furthermore, the effective ductility is larger than the HAZUS ductility factor, \(\mu\), not only for the example S1L building but for all HAZUS building types and design code levels.

With the third option we maintain the HAZUS ductility factor (without the additional HAZUS overstrength factor) by choosing \(\mu \times D_y^*\). It should be noted here that \(\mu \times D_y^*\) is different than the ultimate displacement \(D_u\) that was used in applying the equal area rule to determine \(D_y^*\). Thus, a new yield displacement can be computed based on the equal area rule with \(\mu \times D_y^*\) instead of \(D_u\), via an iterative procedure that can be performed until...
the new yield and ultimate displacements are converged within a specified tolerance. This iteration is left to future work.

Given the new yield capacity point determined in Section 3.1, the ultimate capacity acceleration may be determined in the two ways shown in Figure 4: 1) $\lambda \times A_y^*$, multiplication of the HAZUS overstrength ratio, $\lambda$, and the new yield acceleration, $A_y^*$; 2) use of $A_u = (\lambda \times A_y)$, the HAZUS ultimate capacity acceleration. If we choose $\lambda \times A_y^*$, it results in unreasonably large strain hardening ratio for general buildings. If we choose $A_u$, then $A_y^*/A_y^*$ becomes smaller than the HAZUS overstrength factor $\lambda$, but it results in a more reasonable strain hardening ratio. Also, we know that the ultimate capacity acceleration has large effects on building responses and fragility functions, based on sensitivity analysis (Ryu et al., 2008). Thus, we choose to keep the HAZUS ultimate capacity acceleration, $A_u$, for our multilinear capacity curve.

### 3.3. Residual Capacity Point ($D_r^*$ and $A_r^*$)

As illustrated above in Figure 3, we introduce a residual capacity point, and thereby negative stiffness in the multilinear capacity curve past the ultimate (capping) point, because it has significant effects on the responses in nonlinear dynamic analyses (Ibarra, 2003). With negative post-capping stiffness and various hysteresis models, we may be able to simulate strength and/or stiffness deterioration and collapse behavior. It is difficult, though, to determine the residual capacity point because it is not included in HAZUS and there is not enough guidance or information on the residual capacity for generic types of buildings. Based on the damage state descriptions from HAZUS, the residual capacity displacement (i.e., where the residual strength branch starts), $D_r$, may be determined in the two ways shown in Figure 4: 1) use of $m_{DST\text{ complete}}$, the median damage state threshold from HAZUS for the complete damage state; 2) $(m_{DST\text{ complete}}/m_{DST\text{ extensive}}) \times D_u^*$, multiplication of the ratio of the median damage state threshold for the complete and extensive damage states and the new ultimate displacement determined in the Section 3.2. The resulting negative post-capping stiffness ratios for the two options are -0.10 and -0.15, respectively, which are reasonable values for the example S1L building. If we choose $m_{DST\text{ complete}}$, however, then the resulting negative post-capping stiffness gets shallower from the HAZUS high-code to pre-code design levels, contrary to common sense, instead of getting steeper like it does if we use $(m_{DST\text{ complete}}/m_{DST\text{ extensive}}) \times D_u^*$. Thus, we choose $(m_{DST\text{ complete}}/m_{DST\text{ extensive}}) \times D_u^*$ for the residual capacity displacement of our multilinear capacity curve.

![Figure 7 Capacity curves with various residual capacity points](image-url)
To determine the residual capacity acceleration, $A_r^*$, we perform sensitivity analysis considering residual strength ratios ($A_r^*/A_y^*$) of 0.0, 0.2, and 0.4 based on (Ibarra, 2003). Figure 7 shows the capacity curves for the three different residual capacity accelerations, and Table 2 summarizes the resulting median spectral acceleration capacity from the fragility function computed for each damage state (via the methodology described briefly in Section 1). The effect of the negative post-capping stiffness in the multilinear model can be easily identified in the extensive and complete damage states, but the median spectral acceleration capacities are relatively insensitive to the residual strength ratio for the example S1L building. Thus, one might choose any residual strength ratio in the range considered, say 0.2.

Table 2 Median spectral acceleration capacity for each damage state for the multilinear capacity curves with three different residual strength ratios, compared with those for curvilinear and trilinear capacity curves. (Note that the results in Table 1 for the trilinear capacity curves used the HAZUS, not new, ultimate capacity point.)

<table>
<thead>
<tr>
<th>Damage states</th>
<th>Multilinear ($\lambda_r = 0$)</th>
<th>Multilinear ($\lambda_r = 0.2$)</th>
<th>Multilinear ($\lambda_r = 0.4$)</th>
<th>Curvilinear</th>
<th>Trilinear with new yield and ultimate points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slight</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>Extensive</td>
<td>1.61</td>
<td>1.65</td>
<td>1.67</td>
<td>1.68</td>
<td>1.79</td>
</tr>
<tr>
<td>Complete</td>
<td>3.09</td>
<td>3.23</td>
<td>3.34</td>
<td>3.52</td>
<td>3.77</td>
</tr>
</tbody>
</table>

4. CONCLUSION

To create a more realistic capacity curve model that is easy to implement for nonlinear time history analysis, we construct multilinear capacity curves that are consistent with the curvilinear capacity curves and median damage state thresholds provided in HAZUS. The multilinear capacity curves include negative stiffness past the ultimate (capping) point, which can be used to simulate strength and/or stiffness deterioration and collapse behavior. Our choices for the parameters of the multilinear capacity curve model are as follows: 1) a yield capacity point, $(D_y^*, A_y^*)$, based on an equal area rule relative to the curvilinear capacity curve provided in HAZUS; 2) $D_y^* = \mu \times D_y$ and $A_y^* = A_y$ for the new ultimate capacity point, where $\mu$ and $A_y$ are from HAZUS; 3) $D_y^* = (m_{DST\,complete}/m_{DST\,extensive}) \times D_y$ and $A_y^* = 0.2 \times A_y$ for the residual capacity point, where $m_{DST\,extensive}$ and $m_{DST\,complete}$ are from HAZUS. In choosing these parameters we perform nonlinear time history analyses of the new multilinear SDOF systems with various capacity parameter values and investigate their effects on resulting fragility functions through sensitivity analysis. As an illustration, in this paper we propose appropriate capacity parameter values for a moderate-code low-rise steel moment resisting frame building (labeled S1L in HAZUS) based on the sensitivity analysis results. Although not reported here, the applicability of the procedure proposed in this study has been investigated for other types of structures in HAZUS. In future work, comparisons between the multilinear capacity curves and the curvilinear ones with original parameter values provided in HAZUS will be further extended from fragility functions to risk and loss estimation.

REFERENCES